* **Elem diff bet program and decision problems**

**Programs**

1. **Finite Instructions**:
   * A program is made up of a finite set of instructions, where each instruction can be represented as a finite string of bits.
   * Thus, the program itself is a **finite string of bits**.
2. **Inputs to a Program**:
   * A program typically takes a **finite set of inputs** during execution. Each input can be **very large** but must always be finite in size.
   * **Answer to Your Doubt**:
     + Inputs cannot be infinite. They must be **finite**, even if they are extremely large (e.g., a 1 GB file or a 1,000-digit number).
     + This is because a program must be able to process and store the input within the memory of a computer, which requires the input to have a finite representation.

**Decision Problems**

1. **Definition**:
   * A decision problem is not a specific program. It is a conceptual question that asks for a "yes" (1) or "no" (0) answer for **every possible input** from a given set of inputs.
   * This means a decision problem involves answering for an **infinite set of possible inputs**.
2. **Infinite Input Sequences**:
   * The "range" of inputs for a decision problem is typically infinite. For example:
     + "Is a number even?" needs an answer for every integer: 0,1,2,…0, 1, 2, \dots0,1,2,….
     + "Is a number prime?" needs an answer for all possible integers.
   * The answers to these inputs can be represented as an **infinite string of bits**, where each bit corresponds to a "yes" (1) or "no" (0) for each input.
3. **Key Difference with Programs**:
   * A program handles only a **finite subset of inputs at a time**.
   * A decision problem conceptually defines a solution for an **infinite set of inputs**.

**Clarification of Your Summary**

**Program Section**

* ✅ "Has a set of instructions" → Correct.
* ✅ "An instruction can be represented by a finite string of bits" → Correct.
* ✅ "A program is a finite string of bits" → Correct.
* ✅ "It takes only finite inputs, where each input can be very large" → Correct.
* ❓ "Can an input element be infinite?" → **No, inputs must always be finite.** Inputs can be arbitrarily large but still finite.

**Decision Problem Section**

* ✅ "Has a range of yes (1) or no (0) for each input element" → Correct.
* ✅ "Each input set takes into consideration all possible inputs, which are generally infinite in length" → Mostly correct but needs clarification:
  + The "input set" (the set of all possible inputs) is **infinite**.
  + Each individual input (e.g., a single number) is **finite**.
  + The infinite bit string represents the answers to an infinite number of inputs, not the inputs themselves.

**Summary Table**

| **Aspect** | **Program** | **Decision Problem** |
| --- | --- | --- |
| **Nature** | A finite set of instructions | A conceptual question with "yes/no" answers for all inputs |
| **Bit Representation** | Finite string of bits | Infinite string of bits (answers for all inputs) |
| **Inputs** | Finite, large but finite | Infinite set of all possible inputs |
| **Output** | Finite result for given input(s) | Infinite sequence of answers for all possible inputs |

* Number of Problems to solve more that programs avl to solve them

1. A program operates on a finite subset of inputs during execution, while a decision problem conceptually involves answering for an infinite input set.

* "Finite" means countably finite; "infinite" means countably infinite.

2. Each program processes and produces an output for one input at a time during its execution.

3. The number of decision problems is uncountably infinite, while the number of programs is countably infinite. Thus, there are theoretically more problems to solve than there are programs available to solve them.